A Survey of Alternative Equity Index Strategies[1]

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First Draft: February 24th, 2010
Second Draft: December 28th, 2010
Current Draft: March 3rd, 2011

Abstract

A number of quantitative investment strategies are offered to investors as passive equity indexes. We review methodologies behind the more popular ones and provide an integrated evaluation framework. In our comparison the strategies outperform their cap-weighted counterparts largely due to exposure to value and size factors. Given this insight, these strategies are similar to each other and one can be mimicked by combinations of others. Therefore, implementation cost should be an important evaluation criterion.

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[1] The authors would like to acknowledge Lillian Wu for her most capable research assistance. We also thank Ross Amos (Towers Watson), Robert Arnott, Li-Lan Kuo, Max Moroz, Omid Shakernia, Phil Tindall (Towers Watson), Vivek Viswanathan, Rodney Sullivan (*FAJ* editor), two anonymous *FAJ* referees, and seminar participants at Mercer Consulting, Russell Indexes, Hewitt Consulting, and EDHEC for their comments and assistance. We also express our gratitude to Yves Choueifaty (TOBAM) for clarifications on his paper and his encouraging comments and Noël Amenc (EDHEC) for comments on the potential bias in the paper against the optimized portfolio constructions. We also thank Katrina Sherrerd, Michael Larsen, Joel Chernoff, Jaynee Dudley, and Elizabeth Collins for their editorial assistance.

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Introduction

Recently, a number of alternative approaches to passive equity investing have gained popularity by claiming to offer superior risk-adjusted performance to traditional market-capitalization-weighted indexes.[6] Some of these strategies, such as equal-weighting and minimum-variance, have been around for decades but have only recently garnered meaningful interest. Other approaches, such as Intech's Diversity-Weighted℠ Index, Research Affiliates' Fundamental Index®strategy, QS Investors'[7] Diversification Based Investing (DBI), TOBAM's[8] Maximum Diversification Index®,[9] and EDHEC's Risk Efficient Index are relatively new entrants to the world of passive investing. Arguably, the highly charged debates surrounding the Fundamental Index approach proposed in Arnott, Hsu, and Moore (2005) have spawned much of the recent movement to explore alternative passive equity strategies. If some investors have become convinced that a more intelligent passive equity portfolio is possible, most, however, remain baffled by the available array of options.

The aim of this research is to produce an apples-to-apples comparison of the alternative beta strategies in a controlled backtesting environment with full disclosure on data sources, parameters, and estimation methodologies; in particular, we want to examine the performance characteristics driven by the key assumptions for the strategies rather than by the implementation subtleties of the commercial products. We did not attempt to replicate the actual investment products derived from these strategies, nor to provide investment recommendations on the commercial products based on the strategies; readers must separately research the commercial products as some of them will no doubt claim additional enhancements and refinements to what is reported in this study. Additionally, the commercial products are likely to differ in their asset management fees and expenses, which we do not analyze here. All of the backtests were based on the methodologies disclosed in the public domain, such as in published journal articles or available research papers.

We found that all of the alternative betas examined in this paper outperform their corresponding cap-weighted benchmark index. However, the outperformance is entirely explained by positive exposure to the value and size factors; the alternative betas display no Carhart (or Fama–French) alpha. Additionally, the various strategies can be combined to mimic each other—for example, cash and equal-weighting can be combined to mimic the minimum-variance strategy.[10] This seems to suggest that the different construction methodologies do not actually provide investment insights beyond what is identified in equal-weighting, which outperforms due to its size and value tilts. Nonetheless, these alternative betas are valuable innovations because conventional value and small-cap indexes display

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[6] We are using the language of many of the investment consultants, who now classify quantitative indexes with transparent methodology disclosure as "passive indexes." These strategies are also being referred to as "alternative equity betas," or just "alternative betas."

[7] Formerly DB Advisors, owned by Deutsche Bank.

[8] Formerly Lehman Brother's QAM (Quantitative Asset Management).

[9] The MaxDiv® Index is a long-only version of TOBAM's anti-benchmark strategy based on Choueifaty and Coignard (2008).

[10] See Arnott, Kalesnik, Moghtader, and Scholl (2010) for an example using the Fundamental Index strategy, minimum-variance portfolio, and equal-weighted portfolio to span the three-factor space of market-cap, value, and size.

negative Carhart and Fama–French alphas as documented by Hsu, Kalesnik, and Surti (2010). We conclude that the alternative betas which have lower implementation costs vis-à-vis lower turnover and higher portfolio liquidity could be reasonable options for investors seeking to achieve better performance relative to traditional cap-weighted indexes. Additionally, combining multiple alternative betas allow investors to target the desired levels for market, value, and size tilting in their portfolios.

Methodology

For each alternative beta, we generated a U.S. and a developed global backtest. For U.S. portfolios, we used the merged CRSP/Compustat database and for global portfolios the merged Worldscope/Datastream database. Also, each strategy was backtested with both annual and quarterly rebalancing frequency to observe strategy robustness to varying rebalancing frequency. Portfolios were formed annually (quarterly) on the basis of market price data at the market close on the last trading day each year (quarter).

For both the U.S. and global portfolios, we drew eligible stocks for inclusion from the largest 1,000 stocks.[11] Based on the final inclusion criteria and the portfolio's observed weighted average market-capitalizations, all of the portfolios can be classified as members of the large-cap core category.

Total returns were calculated for each strategy at a monthly frequency from 1964 through 2009 for the U.S strategies, and from 1987 through 2009 for the global strategies. The choice of date ranges depended entirely on the breadth of historical data available to form portfolios. [12]

Investment Strategy Descriptions

The strategies studied can be classified into two categories: (1) heuristic-based-weighting methodologies, and (2) optimization-based-weighting methodologies. Heuristic-based strategies are *ad hoc* weighting schemes established on simple and, arguably, sensible rules. Included in the heuristic category are equal-weighting, risk-clusters equal-weighting, cap-weighting blended with equal-weighting, and weighting by historical financial variables. Optimization-based strategies result from an exercise to maximize the portfolio *ex ante* Sharpe ratio subject to practical investment constraints. In this category are minimum-variance strategies and a variety of maximal Sharpe ratio portfolios based on various expected return assumptions.

In the following subsections, let $\mathbf{x} = \langle x_1, x_2, \ldots, x_N \rangle$ represent a vector of portfolio weights such that the portfolio is fully invested, $\sum_{i=1}^{N} x_i = 1$, and there are no short positions, $x_i \geq 0$ for all N stocks. The notation $\mathbf{x}$ will be used to signify a portfolio. We briefly describe each alternative beta strategy in the next section, focusing on the investment intuition while providing enough detail about portfolio

[11] The various strategies use different definitions of "large." For most, large is measured by year-end market capitalization; for the Fundamental Index strategy, it is measured by company financial variables (e.g., book value).

[12] We acknowledge that our backtested results generally cover a longer time span and more stocks than those used some alternative beta providers. This can result in Sharpe ratios and information ratios which are different from their self-reported performance statistics. We wish to assure the index providers and the readers that the sample ranges were selected to utilize the available data from the CRSP/CompuStat and Worldscope/DataStream databases and not to favor a particular strategy over another.

construction to illustrate the essence of a strategy without belaboring the technical subtleties. For interested readers, the exact construction methodologies and the resulting time series can be obtained from the authors.

Heuristic-Weighting Strategies

Although the focus of the article is empirical, in the following subsections we take some liberty to interpret the investment philosophies underlying the various heuristic-weighting strategies. Specifically, we interpret these strategies as extensions of an equal-weighting strategy—each attempts to eliminate some undesirable portfolio characteristics associated with naïve equal-weighting.

Equal-Weighting: *In an equal-weighted portfolio***,** constituents are selected from the largest $\boldsymbol{N = 1{,}000}$ stocks sorted by descending market-capitalization on the reconstitution date. The weight of each stock is set to 1/N.

A feature of equal-weighting worth noting is that the resulting portfolio risk/return characteristics are highly sensitive to the number of stocks chosen to be included. Although the S&P 500 Index and the Russell 1000 Index have nearly identical risk/return characteristics over time, the equal-weighted S&P 500 and equal-weighted Russell 1000 portfolios have dramatically different risk/return characteristics. Namely, the equal-weighted Russell 1000 is relatively more volatile and has significantly greater exposure to small-cap names.

Risk-Clusters Equal-Weighting: The equal-weighted portfolio strategy is too naïve for some investors. Specifically, the portfolio characteristics are dictated largely by the arbitrary choice of the stock universe on which to apply equal-weighting. The risk-clusters equal-weighting (RCEW) methodology improves upon the simple equal-weighting scheme by equal weighting risk-clusters instead of individual stocks.

The QS Investors' Diversification Based Investing (DBI) is related to the RCEW approach. The DBI methodology has two distinguishing features: (1) It defines risk-clusters based on country and sector membership, and (2) it equally weights the country/sector portfolio within each risk-cluster portfolio. In this study, we examined a representative risk-clusters equal-weighting approach which is comparable to the DBI construction methodology. We note that other adaptations exist in the marketplace, which are valid interpretations of the concept.

At each portfolio reconstitution date, we first acquired monthly time series returns for $L \times M$ country/sector portfolios; these country/sector portfolios are weighted by market-capitalization.[13] U.K. Information Technology, for example, is one such country/sector portfolio. We then specified the desired

[13] For the U.S. application, we used 30 industry sectors as defined by Kenneth French and used the industry portfolio returns from his website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). For the global application, we used all listed stocks tracked in Datastream from 24 developed countries and 12 industry sectors to construct 288 country/sector portfolio returns.

number of risk-clusters, k,[14] and used a standard statistical technique[15] to partition the $L \times M$ country/sector portfolios into k mutually exclusive risk-clusters.[16] To offer some insight, the algorithm identifies correlations between all pairings of country/sector portfolios. Country/sectors that co-move strongly with other country/sector portfolios are gathered into a risk-cluster. Once all k distinct risk-clusters have been identified, country/sector portfolios were then assigned an equal-weight within each risk-cluster. Final portfolios were generated, again, by equal-weighting each of the k risk-clusters. In our backtest, we examined two portfolios with different k's to illustrate how variations of k impact portfolio performance. For the first case, we formed $k = 20$ clusters for the global RCEW portfolio, and $k = 7$ for the United States. For the second case, we used $k = 10$ and $k = 4$, respectively.

The advantage of the RCEW methodology over simple equal-weighting is the robustness of the resulting portfolio to the size of the chosen stock universe. Recall applying simple equal-weighting to the S&P 500 and Russell 1000 results in portfolios with different risk/return characteristics. The RCEW approach, however, would produce portfolios with similar risk/return characteristics.

Diversity-Weighting: Two other potential concerns with the equal-weighting method are relatively high tracking error against the cap-weighted benchmark and excess portfolio turnover. A simple solution is to blend portfolios based on equal-weighting and cap-weighting to attenuate the levels of tracking error and turnover. We introduce diversity-weighting as one of the more well-known portfolio heuristics which blends cap-weighting and equal-weighting.

Fernholz (1995) defined stock market diversity, D_p, as:

$$D_p(\mathbf{x}_{Market}) = \left[\sum_{i=1}^{N}\left(x_{Market,i}\right)^p\right]^{1/p}, p \in (0,1) \quad (1)$$

where $x_{Market,i}$ is the weight of the i^{th} stock in the cap-weighted market portfolio and then defined a portfolio weighting strategy, where portfolio weights were defined as:

[14] The number of risk-clusters may be chosen arbitrarily or by a variety of quantitative methodologies, such as principal component analysis.

[15] Popular clustering methodologies include agglomerative hierarchical clustering, divisive hierarchical clustering, and k-medoid partitioning. See Kaufman and Rousseeuw (1990) for details. Intuitively, the clustering methodology seeks to group similar data items (in our case, equity industry portfolios) together based on some definition of similarity (in our case similarity is defined by correlation—if two portfolios are highly correlated, they are considered to be similar and should be grouped together). Although we observed differences in the resulting portfolios from applying different clustering methods, there are *no ex ante* reasons to believe that one clustering methodology should lead to better portfolios than another methodology. We report only the results based on the k-medoid partitioning method, which is the most robust to outlier effects in data and allows the user to create any number of clusters with ease.

[16] The *dissimilarity* measure between two country/sector time-series returns is computed by $\hat{d}_{i,j} = \sqrt{\frac{1-\hat{\rho}_{i,j}}{2}}$, where $\hat{\rho}_{i,j}$ is the sample return correlation between country/sector portfolios *i* and *j*.

$$x_{Diversity,i} = \frac{\left(x_{Market,i}\right)^p}{\left[D_p(\mathbf{x}_{Market})\right]^P} \quad i = 1, \dots, N, p \in (0,1) \qquad (2)$$

where the parameter p targets the desired level of portfolio tracking error against the cap-weighted index.

Intuitively, a diversity-weighting strategy can be viewed as a method for interpolating between cap-weighting and equal-weighting.[17] Generally, this process redistributes weights from larger names in the cap-weighted portfolio to the smaller names as p moves from 1 to 0. In the extreme case, when $p = 0$, the diversity-weighted portfolio is equivalent to equal-weighting, and when $p = 1$, the diversity-weighted portfolio is equivalent to cap-weighting.

In our replication, we started with the top $N = 1{,}000$ stocks sorted by descending market-capitalization on each reconstitution date, and then formed weights according to equation (3). We backtested two specifications of this strategy, one with $p = 0.76$, which is the parameter chosen by Intech for its U.S. Large Diversity-Weighted Index, and another with $p = 0.50$ to illustrate the effects of the parameter p on portfolio risk/return characteristics.

Fundamentals-Weighting: Arnott, Hsu, and Moore (2005) described a methodology for weighting stock indexes by constituent firms' accounting size, measured by reported financial variables such as total sales and book value. Their aim was to propose weighting measures that are uncorrelated with the firms' market valuations. Hsu (2006) argued if market prices contain non-persistent pricing errors, then portfolios weighted by price-correlated measures, such as market-capitalization, are suboptimal[18]. In this framework, fundamentals-weighting and other price-uncorrelated weighting schemes achieve the same effect as equal-weighting.[19] Arnott, Hsu, and Moore (2005) argued weighting by accounting-based measures of size improves upon an equal-weighting strategy by reducing relative tracking error to the cap-weighted index and turnover, while improving portfolio liquidity and capacity to equal-weighting.

[17] Kaplan (2008) presents a portfolio construction methodology which blends accounting-size-weighting with cap-weighting. This approach can be viewed as similar to the Fernholtz methodology, which blends equal-weighting with cap-weighting. Like the Fernholtz approach, this allows the investor to control the tracking error of the resulting blended portfolio relative to traditional cap-weighted benchmarks.

[18] Perold (2007) challenges the Arnott, Hsu and Moore (2005) findings. First, he pointed out that if prices follow a random walk without mean reversion, then the capitalization weighted index does not experience return drag. Vuolteenaho (2002), among others, document mean reversion in prices, so we maintain our view that mean reversion is a valid assumption. Perold also pointed out that cap weighting would not have a return drag if fair value were to be randomly distributed around the price. This is a valid theoretical argument. However, practically, we believe it is more intuitive to think that the price of the company is distributed around the fair value, not the other way around.

[19] Arnott, et al. claimed that the exact choice of weighting metrics and the number of financial metrics chosen do not result in statistically different performances in the long run.

We constructed a representative fundamentals-weighting approach that replicated the Arnott, Hsu, and Moore (2005) four-factor Fundamental Index methodology.[20] For this research, the four accounting size metrics are defined as the past five years' average sales, five years' average cash flow, five years' average total dividends paid, and the past year's book value. For each fundamentals-weighting portfolio, constituents were sampled from the largest $N = 1{,}000$ companies sorted by descending accounting size. The portfolio weight of the i^{th} stock is defined by:

$$x_{Accounting\ Size,i} = \frac{Accounting\ Size_i}{\sum_{i=1}^{N} Accounting\ Size_i} \qquad (3)$$

The final Fundamental Index portfolio is then constructed by averaging the sales, cash flow, dividends, and book value-weighted portfolios. Additionally, we also compute a portfolio based on the prior year dividend to illustrate the effect of using a single unsmoothed accounting variable.[21] All accounting data represent annual financial performance and are lagged two years to prevent look-ahead bias.

Optimization-Based-Weighting Strategies

In theory, mean-variance optimization is a fantastic way to form passive portfolios, yet it frequently falls short of its aim when applied in practice. The two inputs required to generate a mean-variance optimal portfolio—expected returns for all stocks and their covariance matrix, are notoriously difficult to estimate.

Chopra and Ziemba (1993) showed that if an investor's return forecast contains errors, even if the errors are small in magnitude, the performance of the resulting mean–variance optimization (MVO) may be meaningfully reduced. Expected returns for individual stocks are very difficult to forecast accurately. The return covariance matrix, given its size, is also difficult to estimate.[22] Michaud (1989) demonstrated that MVO can actually magnify errors in the empirical covariance matrix by over(under)-emphasizing assets with small (large) estimated variances and covariance. In the following subsections, we introduce several alternative-beta strategies applying MVO. Each attempts to overcome obstacles associated with forecasting risks and return for a large number of stocks.

Minimum-Variance: Because return forecasting is so difficult and the potential for errors so large, Chopra and Ziemba (1993) suggested that portfolio outcomes could be improved by assuming all stocks have the same expected returns. Under this seemingly stark assumption, the optimal portfolio is the

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[20] FTSE RAFI Index Series, FTSE GWA Index Series, Russell Fundamental Index Series, MSCI Value Weighted Indices, and WisdomTree dividend and earnings weighted indexes use accounting-based measures of size in their methodologies. However, they employ different weighting variables and use different selection universes, which lead to different in-sample performances and can lead to different out-of-sample returns. We believe our replication captures the salient features for these indexes and provide a valuable reference point. For a more detailed comparison between these strategies, please see the Research Affiliates Working Paper "What Makes Fundamental Index Methodology Work?" by Hsu and Kalesnik.

[21] This choice is motivated by the WisdomTree dividend-weighted ETFs.

[22] The size of the covariance matrix for a 1,000-stock portfolio is 1000 × 1000, with 500,500 unique parameters that must be estimated.

minimum-variance portfolio. Haugen and Baker (1991) and Clark, de Silva, and Thorley (2006) demonstrated that minimum-variance strategies improve upon their cap-weighted counterparts in historical backtests—supplying better returns with reduced volatility. Note that a minimum-variance portfolio is mean-variance optimal only if stocks are assumed to have the same expected returns; while the minimum-variance portfolio is unlikely to be mean-variance optimal, it does appear that mean-variance optimality is not required to deliver outperformance against standard cap-weighted indexes.

To construct a minimum-variance strategy, on each reconstitution date we selected the largest $N = 1{,}000$ companies sorted by descending market-capitalization. The covariance matrix was estimated by using monthly excess returns for the prior 60 months.[23] To reduce the influence of outliers in the empirical covariance matrix, we used a shrinkage estimator similar to the method of Ledoit and Wolf (2004); we also considered other covariance estimation techniques as tests for strategy robustness. Portfolio weights for a minimum-variance strategy can be expressed as the solution to the following optimization problem:

$$\min_x x'\hat{\Sigma}x \;\; \textit{subject to} \begin{cases} \Sigma_{i=1}^{N} x_i = 1 \\ l \leq x_i \leq u \end{cases} \forall i \tag{4}$$

where $\mathbf{x}$ is the vector of portfolio weights and $\hat{\Sigma}$ is the estimated covariance matrix. We enforced no short-selling by setting a lower bound $l = 0$. Additionally, we included a position cap of $u = 5\%$ to avoid excess concentration of weight in any given stock.

Maximal Sharpe Ratio I: Given that stocks are not likely to all have the same expected returns, the minimum-variance portfolio, or any practical expression of its concept, is theoretically unlikely to be the portfolio with the maximum *ex ante* Sharpe ratio. To improve upon a minimum-variance strategy, investors need to incorporate useful information on future stock returns.

Choueifaty and Coignard (2008) proposed a simple linear relationship between the expected premium, $E[R_i] - R_f$, for a stock and its return volatility, σ_i:

$$E[R_i] - R_f = \gamma\, \sigma_i \;\forall i, \tag{5}$$

where $\gamma > 0$. Under this assumption, the constrained Sharpe ratio optimization problem can be written as:[24]

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[23] We impose a requirement that at least 60 months of past return be available for the stock to be included in our simulation. We could also use daily data to estimate the covariance matrix. In our research we do not find using daily data to create materially different backtested results.

[24] Choueifaty and Coignard (2008) proposed an extremely clever and efficient way to perform the equivalent constraint maximization. They created synthetic securities by, first, dividing the original time series returns for a stock by the estimated volatility. Then, they could compute the optimal Sharpe ratio portfolio as a minimum-variance portfolio, which is an easier computational exercise. One could chose to solve the optimal Sharpe ratio problem directly; however, the computational complexity becomes significantly greater.

$$\max_{x} \frac{\mathbf{x}'\hat{\sigma}}{\sqrt{\mathbf{x}'\hat{\boldsymbol{\Sigma}}\mathbf{x}}} \text{ subject to } \begin{cases} \Sigma_{i=1}^{N} x_i = 1 \\ l \leq x_i \leq u \end{cases} \forall i \quad (6)$$

where x is the vector of portfolio weights, $\hat{\Sigma}$ is the estimated covariance matrix, and $\hat{\sigma}$ is the vector of estimated returns volatilities. This encapsulates the general framework we used to compute a proxy portfolio for TOBAM's Maximum Diversification Index.

In the backtests, portfolio constituents were selected from the largest $N = 1{,}000$ stocks sorted by descending market-capitalization on the reconstitution date. We enforced no short-selling with $l = 0$; following Choueifaty and Coignard (2008), we also let $u = 10\%$ to limit excess concentration of weight in any given position. The covariance matrix and volatilities were estimated using the same shrinkage technique described earlier to handle estimation errors.

As an aside, equation (6) represents a departure from standard finance theory, which states that only the non-diversifiable component of volatility (systematic risk) should earn a premium. We note that equation (6), when interpreted to apply to stocks and portfolios of stocks, becomes internally inconsistent; it suggests that all stocks and portfolios should have the same Sharpe ratio, and therefore, volatilities are linearly additive in equilibrium, which cannot be correct.[25] We also note that there exists conflicting empirical evidence for the positive relationship between diversifiable risk and expected return. Goyal and Santa-Clara (2003) found a positive relationship, whereas Ang, Hodrick, Xing, and Zhang (2006, 2009) found a negative relationship.

Maximum Sharpe Ratio II: A related portfolio approach developed by Amenc, Goltz, Martellini, and Retkowsky (2010) assumes the expected returns for a stock are linearly related to its downside semi-volatility. Amenc, et al. argue that investors are more concerned with portfolio losses than gains. Thus risk premium should be related to downside volatility as opposed to volatility. This assumption serves as a foundation for the EDHEC Risk Efficient Index.

To demonstrate how this strategy is constructed, the downside semi-volatility for the $\boldsymbol{i^{th}}$ stock is defined as:

$$\delta_i = \text{Downside Semi-Volatility}_i = \sqrt{E\left[\min\left(R_{i,t}, 0\right)^2\right]} \quad (7)$$

where $R_{i,t}$ is the return for stock i in period t. Under this assumption, the traditional MVO problem of maximizing a portfolio's Sharpe ratio can be expressed as:

[25] Choueifaty pointed out to us that in his paper he meant for equation (6) to only apply to stocks and not to portfolios. In a new paper Choueifaty, Froidure, and Reynier (2010), Choueifaty argued that if risky assets have expected returns proportional to its diversification ratio × volatility, then volatility would not need to be additive.

$$\max_{\mathbf{x}} \frac{\mathbf{x}'\widehat{\boldsymbol{\delta}}}{\sqrt{\mathbf{x}'\widehat{\Sigma}\mathbf{x}}} \text{ subject to } \begin{cases} \Sigma_{i=1}^{N} x_i = 1 \\ l \leq x_i \leq u \end{cases} \forall i \tag{8}$$

where $\mathbf{x}$ is the vector of portfolio weights, $\hat{\Sigma}$ is the estimated covariance matrix, and $\hat{\delta}$ is the vector of estimated downside semi-volatilities.

Amenc, Goltz, Martellini, and Retkowsky (2010) use a two-stage estimation heuristic for estimating the semi-volatility for stocks. The method first computes empirical semi-volatilities and sorts stocks by these estimates into deciles. It then sets the semi-volatility for stocks in the same decile equal to the median value of the containing decile.[26] This methodology also imposes strong restrictions on single-stock weights, with a lower limit of $l = 1/(\lambda N)$ and upper limit $u = \lambda/N$. Amenc, et al. use $\lambda = 2$; at $\lambda = 2$, portfolio weights are restricted to vary between bounds of 0.05% and 0.2%. These position constraints have the effect of shrinking the unconstrained Risk Efficient Index portfolio weights toward equal-weighting; note that as λ tends toward 1, the position constraints approach 1/N, or equal-weighting. To assess the impact of the λ restriction, we backtested one portfolio (with $N = 1{,}000$ largest capitalization stocks) with $\lambda = 2$ and an additional portfolio with $\lambda = 50$, restricting portfolio weights to lie between 0.002% and 5%, which is more comparable to the other methodologies. In the backtests, we additionally implemented a turnover restriction required by Amenc, et al. which suppresses rebalancing on reconstitution if weights have not deviated significantly from the new model weights; we acknowledge that we were unable to measure the impacts of the various external restrictions separately from the key underlying investment philosophy, which assumes that stock returns should be related to downside volatility.

Empirical Results and Discussions

In this section, we analyze the simulated time series returns for the alternative betas described in the previous section. To assess the accuracy of our simulations, we compared our backtested time series results with the time series results provided by the strategy provider, when available, or with the time series summary statistics reported in published papers. Unless otherwise stated, reported returns are annualized and geometric. We stress, again, that the aim of the backtesting was not to replicate the commercial products being marketed by various asset managers; our aim was to illustrate the performance associated with the key investment philosophies stated in these alternative betas.[27] We provide a careful comparison of these alternative-beta concepts using identical datasets and stock universes, identical rebalancing dates and frequency, identical optimization algorithms, and identical estimation methods for distributional characteristics. In this controlled environment, we can understand how the core features and philosophies of the considered methodologies influence portfolio performance. We are also able to ascertain how often overlooked details of the methodologies such as

[26] This approach is a simple but extreme form of the Bayesian shrinkage technique.

[27] EDHEC has asked us to use its reported time series instead of replicating them, noting that the actual products include carefully calibrated parameters, constraints, and other thoughtful designs that our replication does not properly capture. We acknowledge these concerns and reiterate our disclaimer that commercial products may differ from our simulated results for a variety of reasons.

rebalancing frequency, the number of constituents included, and position constraints influence a strategy's performance.

Universe Construction and Stock Selection Rule

For the U.S. equity universe, we used the CRSP universe, which includes all listed stocks on the NYSE, AMEX, and NASDAQ. We excluded all ETFs, title records, and ADRs. CompuStat was used for U.S. company financial reporting data; we excluded firms not incorporated in the United States. We then excluded firms without two full years of reported financials or stock return data. For the global universe, we used the Datastream stock universe and constrain the country definition to match the MSCI developed countries classification. The country of origin for a given firm is determined by a variety of factors, namely the location of its primary operations, head offices, incorporation, auditing location, and stock exchange where its shares are most liquidly traded.[28] Again, all ETFs and ADRs are excluded. We used the Worldscope database for global company financial reporting data. For inclusion in a strategy where market-capitalization is used as a discriminating variable, we used the market-capitalization tied to a firm's primary share class prior to rebalancing; we did not float-adjust the market-capitalization. While our constituent universe and stock selection rules do not match the S&P and MSCI rules exactly, we do not expect this to bias our results.

Standard Performance Characteristics

We report summary statistics of the time series returns based on parameters that are most similar to the official specifications provided by the provider of the methodology. We have, however, synchronized the rebalancing time and frequency[29] in our tests as well as on the constituent universe to ensure that we have a controlled environment for performance comparison. For U.S. portfolios, monthly total returns were calculated for each portfolio strategy from 1964 through 2009, and for global portfolios from 1987 through 2009. Unless otherwise specified, we use annual rebalancing on the last day of the calendar year. We report the performance characteristics for the alternative betas in **Table 1a and Figure 1a** for global strategies and in **Table 1b and Figure 1b** for U.S. strategies.

Note that all of the strategies in this study produced meaningfully higher returns than their cap-weighted benchmark over the full sample period. Because we studied only strategies that have achieved commercial or publication success, the observed in-sample outperformance may be explained by

[28] Firms like Accenture and Schlumberger, which are incorporated offshore, with major businesses operating in theUnited States and are most liquidly traded in the United States, were considered U.S. listed companies in our global study; however, they are excluded by the CompuStat database because they do not file financials in the United States. As a result, these firms are not included in our U.S. study.

[29] For the Risk Efficient strategy, we follow the turnover control proposed by Amenc, et al. The annually "rebalanced" Risk Efficient portfolio is reassessed quarterly with the weights overhauled if the portfolio weights deviate from targets by a predefined threshold. Empirically, this quarterly reassessed efficient index rebalances approximately once a year. The "quarterly rebalanced" Risk Efficient strategy in the subsequent section was constructed by raising the reassessing frequency and lowering the rebalancing threshold.

selection bias.[30] To verify the robustness of the results, we report the sub-sample period returns for the strategies in **Appendix I** and the parameter variation analyses in **Appendix II**. We do not find evidence that the long horizon results are dominated by one particular sub-sample period. We also find that all variations in strategy specification continue to deliver outperformance over the cap-weighted benchmark, suggesting that these alternative betas do indeed provide a reliable avenue for improved performance. We will explore the potential sources for the observed outperformance later when we present the Carhart and Fama–French factor analyses.

As advertised, the minimum-variance portfolios show the lowest volatilities of all strategies considered. In general, the optimized strategies tend to have higher tracking errors and lower volatilities; the heuristic-weighting strategies, in contrast, tend toward relatively higher volatilities and lower tracking errors. In the absence of portfolio optimization, most long-only portfolios naturally tend toward a market beta of 1, whereas optimized portfolios naturally tend to give low or zero weights to higher beta stocks, resulting in a portfolio beta significantly below 1. The risk efficient portfolio construction with position constraints (between 0.05% and 0.2%) is the exception; it has empirical market betas of 0.94 (global) and 1.0 (United States) over the full sample periods. This is largely driven by the strategy's imposed position constraints, which tilt the final portfolio weights toward equal-weighting.

With the exception of the RCEW strategy, the heuristic-based portfolios realized significantly lower turnover than the optimized portfolios. This is related to the observation that MVO can be quite sensitive to small changes in estimates of expected returns and covariances, resulting in volatile portfolio weights from one rebalancing to the next. The RCEW methodology involves a quasi-optimization when partitioning country/sector pairings into clusters; this can lead to a relatively higher turnover as a result. It is not clear that portfolio optimization leads to a meaningfully higher degree of diversification than naïve diversification. The full sample period Sharpe ratios from a number of the heuristic-based strategies are comparable to, or greater than, those of the optimized strategies. From the factor analyses in the next section, we find evidence that the reduced volatility of optimized portfolios is driven by a reduction in the exposure to the market beta, rather than a further reduction in idiosyncratic volatility.

Robustness of Strategies

We examined the robustness of the various alternative strategies to changes in some of the "user" and "technical" parameters. Specifically, we consider the total return in excess of the benchmarks for: (1) rebalancing quarterly (instead of annually), (2) constructing portfolios with the 500 largest stocks (instead of the 1,000), and (3) imposing 5% and 1% limits on single-stock concentration. We compare the impact on the optimized portfolios from using different techniques for estimating the return covariance matrix. Generally, we found that strategy variants can often have different in-sample performances;

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[30] Phil Tindall at Towers Watson suggests that the selection bias problem can be addressed by considering a strategy, which we have no *ex ante* knowledge on its historical performance and one which we could not accidentally datamine its control parameter for in sample outperformance, such as weighting by random weights.

however, *ex ante*, we cannot conclude whether one particular variant would have stronger out-of-sample performance.[31]

Table 2 shows that a strategy's performance does not materially depend on the rebalancing frequency. However, quarterly rebalancing increases the turnover nearly twofold.[32] Although some of the alternative-beta strategy vendors employ quarterly rebalancing, this comparison does not find any benefit from more frequent rebalancing, and the elevated turnover will *prima facie* erode performance in implementation.

Reducing the constituent universe from the top 1,000 largest stocks to 500 systematically reduces the performance for every tested strategy. This result is intuitive; for the heuristic-weighting strategies, a narrower universe of larger stocks reduces exposure to the size (small-cap) factor, which reduces portfolio returns over time. For the optimized strategies, the smaller universe diminishes the opportunity set, which reduces performance.

Enforcing a 5% limit on single-stock concentration does not have a significant impact on portfolio return and risk characteristics. As the limit becomes more aggressive, strategy portfolio weights begin to converge toward the equal-weighting. It is useful for investors to understand whether a strategy's performance is driven by the investment philosophy or by external constraints and to be cognizant of when external constraints are overriding the investment methodology.

Tables 3a,b show that the covariance estimation methodology can have a significant impact on the optimized portfolio performance. We used the shrinkage method proposed by Clarke, de Silva, and Thorley (2006) [Bayesian Shrinkage A] and by Ledoit and Wolf (2004) [Bayesian Shrinkage B] in the tables. We also use a principal components analysis method as a third alternative for estimating the return covariance matrix. *Ex ante*, investors have no reason to expect one of the three techniques to offer better portfolio performance than the other. What the tables do demonstrate is that optimized strategies are quite sensitive to the covariance matrix estimation technique employed. Investors should understand this sensitivity when examining backtests associated with optimized portfolios.[33] We note that the same criticism applies to all strategies considered in this paper; variations in parameters and specifications lead to different levels of outperformance. The more sensitive a methodology is to trivial perturbation in parameters or specifications, the less reliable the methodology's backtested outperformance.

Four-Factor Risk-Adjusted Performance

[31] We provide what we consider to be the most interesting variations in the text and appendices. Additional variations can be requested from the authors.
[32] Turnovers are not shown in **Table 2** for the purpose of cleaner presentations; it is intuitive that frequent rebalancing leads to higher turnover. Interested readers can obtain the turnovers, volatilities, and other performance measures from the authors.
[33] We note that the risk efficient portfolio is the exception; its results are relatively immune to changes in the covariance estimation. We suspect that this is, again, driven by the imposed stock weight constraints.

In **Tables 4a**,**b**, we provide the results of a Carhart (1997) four-factor return decomposition for the various strategies.[34] After the strategies are adjusted for market, size, value, and momentum factor loadings, only one strategy, the global Fundamental Index strategy, displays a statistically significant, positive alpha at the 5% level. We note that the U.S. Fundamental Index strategy does not show a significant alpha. Because only 1 out of the 20 strategies/variants tested show a significant alpha, we interpret the alpha as an outlier in the experiment.[35] Almost all of the examined strategies display positive and significant exposure to the size and value factors. We conclude that all strategies examined here outperform because of the positive value and size loadings. In a way, none of these strategies are different from naïve equal-weighting in their investment insights. In the appendix we show the same analysis using the Fama–French three-factor model and reach a similar conclusion.

The absence of any Carhart four-factor alpha is not a surprising result. None of the considered strategies employ nonpublic information, contain useful or uncommon insights, or deliberately seeks exposure to *other* factors/anomalies. We conclude that the investment insights specified in the various strategies, such as expected returns, are linearly related to volatility (Choueifaty and Coignard, 2008), or to downside semi-deviation (Amenc, Goltz, Martellini, and Retkowsky, 2010), do not offer a comparative returns advantage to traditional quant factor-tilting. Recall that for a well-diversified portfolio, volatility is approximately the portfolio's market beta multiplied by the market portfolio's volatility. From the linear relationship that we observe between the market beta reported in **Tables 4a,b** and portfolio volatility reported in **Table 1**, lower portfolio volatility (i.e., as provided by the minimum-variance strategy) results largely from a lower market beta. Whether portfolio optimization measurably improves diversification over the naïve diversification strategy of holding a large number of stocks is unclear.

These alternative betas can still be valuable to investors, even in the absence of statistically significant Carhart-alpha. They provide access to the size and value premia, and should be judged on the basis of their ability to access these factors efficiently. Hsu, Kalesnik, and Surti (2010) found traditional value and small-cap indices exhibit negative Fama–French alphas, suggesting that they may be suboptimal portfolios for providing value and small-cap tilts.[36] Fama–French factor portfolios are also difficult to invest in because they require shorting, experience high turnover at rebalancing, and contain many illiquid stocks. Alternative-beta strategies, which provide efficient long-only access to value and size factors, represent improvements over existing value and small-cap indexes. Insofar as we can

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[34] The Carhart four-factor approach uses the market factor, the Fama–French size and value factor in conjunction with a momentum factor. The U.S. Fama–French size and value factors were downloaded from Kenneth French's website. The global Fama–French factors were created by the authors following the methodology described on French's website. The market factor used in our analysis is the cap-weighted benchmark return minus the risk-free rate, R_f. The momentum factors are constructed based on the methodology described in Carhart (1997). Cremers, Petajisto, and Zitzewitz (2008) claim that the Fama–French three-factor and Carhart four-factor analysis can have downward biases in alpha estimation. We do not address this finding in our empirical analysis.

[35] A Bonferroni correction for multiple comparisons would fail to reject the null hypothesis that the alpha is zero.

[36] Hsu, Kalesnik, and Surti (2010) attribute the negative Fama–French alpha for traditional style indices to the cap-weighting construction, where the more expensive value stocks and small stocks take up larger weights than the cheaper value and small stocks.

efficiently apply alternative-beta portfolios as tools to improve an investor's portfolio Sharpe ratios and information ratios, these strategies are valuable.

Finally, we explored why these alternative betas result in value and small-cap biases. First, most of the strategies load significantly on the value factor. This is consistent with the finding of Arnott and Hsu (2008) and Arnott, Hsu, Liu, and Markowitz (2010), where any portfolio which rebalances regularly toward non-price-based weights naturally incurs a positive value loading. We also find high small-cap loading for diversity-weighting and RCEW, which are more directly related to equal-weighting and for risk-efficient weighting, which is indirectly related to equal-weighting through the weight constraints. This is also not surprising because equal-weighting, by construction, systematically over-weights smaller stocks relative to the comparable cap-weighted index. Additionally, we note that optimized strategies generally have a loading on the market portfolio much less than 1. MVO tends to favor stocks with low average covariance to other stocks in the selection universe (unless the expected return for stocks depends on covariance, as in the CAPM); therefore, MVO often results in large weights to lower-beta stocks and, therefore, a "low-beta" portfolio.

Alternative-Beta Portfolios and Mimicking Portfolios

Each alternative-beta strategy offers different performance advantages over another, but none dominates in all categories. An alternative-beta portfolio represents a mapping to the market, value, and size factors—that is the Fama–French three-factors span the investment opportunity set for the alternative betas—as such they can generally be linearly combined with each other (and/or cash) to mimic one another. As an illustration, we replicated the minimum-variance portfolio with other alternative betas. The minimum-variance portfolio is attractive because of its high Sharpe ratio; historically, it offers a higher return and lower volatility than the cap-weighted benchmark. From the Carhart four-factor decomposition, the minimum-variance portfolio is *special* in its low market beta loading, it suggests that we can construct a minimum-variance-mimicking portfolio by holding x percent in an alternative-beta portfolio and (1 – x percent) in cash, where x is set to ensure that the mimicking portfolio has the same volatility as the minimum-variance portfolio. In **Tables 5a**,**b**, we show the characteristics of the various minimum-variance-mimicking portfolios.

For the global comparison, mimicking portfolios based on RCEW and fundamentals-weighting outperform the actual minimum-variance portfolios as measured by the Sharpe and information ratios. For the U.S. results, maximum-diversification-weighting has nearly comparable performance to the minimum-variance strategy. Even if the performances were similar but not statistically better, these mimicking-portfolios might be considered as more capital-efficient alternatives to traditional minimum-variance portfolios because they free up cash for other investments.

Turnover, Trading Costs, Capacity, and Liquidity

In addition to contrasting strategy performances, we also examined the cost of rebalancing associated with the alternative betas. We compute a number of portfolio characteristics, such as turnover and average bid–ask spread, which are related to the portfolio transaction cost. The results are shown in

Table 6. Turnover measures only one part of the total rebalancing cost; stocks with different liquidity characteristics generally incur different trade costs. In **Table 6**, we report the estimated annual portfolio turnover costs using a transactions cost model proposed by Keim and Madhavan (1997).[37] The trade cost estimate and reported is based on a $100M portfolio; larger portfolios will incur larger percentage costs as buy and sell orders start to approach the average daily trading volume for some of the smaller and less liquid stocks. Strategies with higher weighted average daily trade volume would likely face a smaller cost increase at a larger asset level. Due to the lack of stationarity in the Keim and Madhavan calibrated model (a problem with other calibrated cost models as well), there is potential downward bias in the results for more recent trades (and upward bias to the earlier trades in the sample); we consider our cost estimate results as rough references only. Investors are recommended to separately retain independent execution service providers to perform transaction cost estimation on the alternative betas of interest.[38]

Note that the trading costs estimates are naturally the lowest for market-cap and are economically higher for the other strategies. However, from our estimation, the transactions cost for most strategies generally do not erode the entire relative return in excess of benchmark; we stress that we do not factor in asset management fees and expenses as we do not have such information. Diversity-weighting and fundamentals-weighting generally have the lower annual turnover and trading cost. Their turnover levels are half of the other heuristic portfolios and one-third of the optimized portfolios. The estimated transaction costs for fundamentals-weighting show similar advantage over other strategies, while the costs of diversity-weighting ($p = 0.76$) are only slightly higher than cap-weighting. We wish to point out that the commercial products are likely to have additional turnover constraints and management during rebalancing to address the excess turnover associated with alternative betas; these are not captured in our study.

Because the trading costs are estimated on a $100M portfolio which may not be representative for larger investors, we also provide other portfolio characteristics to illustrate investability and potential transaction costs for more sizeable portfolios. In **Table 6**, we show each portfolio's investment capacity by computing the weighted average market-capitalization and each portfolio's liquidity by examining its weighted average bid–ask spread and daily trade volume. The average market-capitalization measure is representative of the portfolio capacity and gives investors a rough approximation of the dollar

[37] Keim and Madhavan estimated transaction costs using order-level data from US$83 billion worth of equity transactions initiated by various institutional traders. Their model accounts for costs associated with types of trades (buys or sells), style of investments (indexed vs. active; we fix our trades as indexed in this study), price per share, market capitalization, size of trade, as well as exchanges (NASDAQ is more expensive than NYSE). We use their model to estimate trading costs at stock-by-stock level and then aggregate them to obtain portfolio level estimate. Because Keim and Madhaven's model was estimated only for U.S. trades, we modified the model to adjust for additional charges for the London exchange (50 bps for selling) and the Hong Kong exchange (10 bps for both buying and selling). We acknowledge that the model is likely to be more robust for estimating trade costs for U.S. equity transactions than for global transactions.

[38] Since the Keim and Madhaven model isn't stationary (because it takes market-capitalization and price per share from 1992–1993 as factors), its results can be downward biased for estimating costs on portfolio schemes using more recent data. We attempt an adjustment to correct partially for the stationarity issue by adjusting the market-capitalization of stocks to the 1992 level. Using this adjustment with a US$500M portfolio, we found only insignificant increase in estimated portfolio transaction costs.

investment a strategy can accommodate. Fundamentals-weighting and diversity-weighting have twice to three times the average market-capitalization of other alternative betas, and by this measure the greatest portfolio capacities; this means that they will face relatively smaller increases in portfolio transaction cost as investors deploy the strategy at larger asset levels. Fundamentals-weighting and diversity-weighting also generally have the lower bid–ask spreads and higher average daily trade volumes. By construction, fundamentals-weighting uses accounting-size variables that are co-integrated with market-capitalization, accordingly, emphasizing larger capitalization companies. Diversity-weighting is parameterized to partially mimic cap-weighting, which naturally allocates more to larger capitalization companies. Because capitalization is correlated in the cross-section with narrow bid–ask spread and high daily trade volume, unsurprisingly, diversity-weighting and fundamentals-weighting score well on these three liquidity measures.

Conclusion

In this paper, we provided empirical evidence that the popular alternative betas outperform cap-weighted indexing unadjusted for risk factors. Using the Carhart four-factor model, the sources of outperformance were explained by exposures to the value and size factors with risk-adjusted alpha being not statistically different from zero. Because these strategies are essentially spanned by the same return factors (market, value, and size), they can be carefully combined to mimic one another. This finding led us to conclude that, despite the claimed unique investment insight and/or technological sophistication by the purveyors of these strategies, the performances are directly related to a naïve equal-weighting strategy, which produces outperformance by tilting toward value and size factors. Nonetheless, the alternative betas represent efficient and potentially low-cost means to access the value and size premiums because traditional style indices tend to have negative Fama–French alpha, and direct replication of Fama–French factors is often impractical and costly. Additionally, combining alternative betas together (and with cash and equity index futures) would allow investors to target desired levels of value and size tilt in a more optimal fashion in their equity allocation.

As an aside, we point out that mean-variance portfolio optimization, which is required for a number of the alternative betas, does not appear to result in a meaningful diversification improvement over non-optimized portfolios, despite the added complexity. Investment insights about the relationship between stock's expected excess returns and volatility, or downside semi-volatility, do not appear to produce performance benefits that are otherwise not present in other simple portfolio heuristics, which are derived from equal-weighting. This finding is consistent with the long literature documenting the puzzling underperformance of MVO approaches.

Lastly, we caution investors to pay special attention to the potential implementation costs associated with these alternative betas relative to the cap-weighted benchmark. The excess turnover, reduced portfolio liquidity, and reduced investment capacity in addition to the fees and expenses associated with managing a more complex index portfolio strategy may erode too much of the anticipated performance advantage. Because less costly alternative betas may be combined to mimic the more costly alternatives, implementation cost should be one of the evaluation criteria.

References

Amenc, Noël, Felix Goltz, Lionel Martellini, and Patrice Retkowsky. 2010. "Efficient Indexation: An Alternative to Cap-Weighted Indices." *EDHEC-Risk Institution Publication* (January).

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. "The Cross Section of Volatility and Expected Returns." *Journal of Finance*, vol. 51, no. 1 (February):259–299.

———. 2009. "High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence." *Journal of Financial Economics*, vol. 91, no. 1 (January):1–23.

Arnott, Robert D., Jason C. Hsu, and Philip Moore. 2005. "Fundamental Indexation." *Financial Analysts Journal*, vol. 61, no. 2 (March/April):83–99.

Arnott, Robert D., and Jason C. Hsu. 2008. "Noise, CAPM and the Size and Value Effects." *Journal of Investment Management*, vol. 6, no. 1 (First Quarter):1-11.

Arnott, Robert D., Jason C. Hsu, Jun Liu, and Harry Markowitz. 2010. "Can Noise Create the Size and Value Effects?" Working paper, University of California at San Diego and Research Affiliates.

Arnott, Robert D., Vitali Kalesnik, Paul Moghtader, and Craig Scholl. 2010. "Beyond Cap Weight: The Empirical Evidence for a Diversified Beta." *Journal of Indexes*, vol. 13, no. 1 (January/February):16–29.

Barclay's Capital. 2008. "Prise II. The Barclay's Capital Pre-Trade Implementation Shortfall Estimation Model." White Paper.

Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." *Journal of Finance*, vol. 52, no. 1 (March):57–82.

Chopra, V.K., and W.T. Ziemba. 1993. "The Effect of Errors in Means, Variances and Covariances on Optimal Portfolio Choice." *Journal of Portfolio Management*, vol. 19, no. 2 (Winter):6–11.

Choueifaty, Yves, and Yves Coignard. 2008. "Toward Maximum Diversification." *Journal of Portfolio Management*, vol. 35, no. 1 (Fall):40–51.

Choueifaty, Yves, Tristan Froidure, and Julien Reynier. 2010. "Maximum Diversification Portfolio Properties." Working paper.

Clarke, Roger G., Harindra de Silva, and Steven Thorley. 2006. "Minimum-Variance Portfolios in the U.S. Equity Market." *Journal of Portfolio Management*, vol. 33, no. 1 (Fall):10–24.

Cremers, Martijn, Antti Petajisto, and Eric Zetzewitz. 2008. "Should Benchmark Indices Have Alpha? Revisiting Performance Evaluation." Yale School of Management and Dartmouth College working paper.

DiMiguel, Victor, Lorenzo Garlappi, and Raman Uppal. 2009. "Optimal Versus Naïve Diversification: How Inefficient is the 1/N Portfolio Strategy?" *The Review of Financial Studies*, vol. 22, no. 5 (May):1915–1953.

Fernholz, Robert. 1995. "Portfolio Generating Functions." Working paper.

"FTSE Diversification Based Investing Index Series." 2010. FTSE DBI Index Factsheet.

Goyal, Amit, and Pedro Santa-Clara. 2003. "Idiosyncratic Risk Matters!" *Journal of Finance*, vol. 58, no. 3 (June): 975–1007.

Haugen, Robert A., and Nardin L. Baker. 1991. "The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios." *Journal of Portfolio Management*, vol. 17, no. 3 (Spring):35–40.

Hsu, Jason C. 2006. "Cap-Weighted Portfolios Are Sub-Optimal Portfolios." *Journal of Investment Management*, vol. 4, no. 3 (Third Quarter):1–10.

Hsu, Jason C., Vitali Kalesnik, and Himanshu Surti. 2010. "An Examination of Traditional Style Indices." *Journal of Index Investing*, vol. 2, no. 2 (Fall):14–23.

Jagannathan, Ravi, and Tongshu Ma. 2003. "Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps." *Journal of Finance*, vol. 58, no. 4 (August):1651–1684.

Kaplan, Paul D. 2008. "Why Fundamental Indexation Might—Or Might Not—Work." *Financial Analysts Journal*, vol. 64, no. 1 (January/February):32-39.

Kaufman, Leonard, and Peter J. Rousseeuw. 1990. *Finding Groups in Data: An Introduction to Cluster Analysis*. New York: Wiley-Interscience.

Keim, Donald and Ananth Madhavan. 1997. "Transaction Costs and Investment Style: An Inter-Exchange Analysis of Institutional Equity Trades." *Journal of Financial Economics*, vol. 46, no. 3 (December):265–292.

Ledoit, Olivier, and Michael Wolf. 2004. "Honey, I Shrunk the Sample Covariance Matrix." *Journal of Portfolio Management*, vol. 30, no. 4 (Summer):110–119.

"Methodology for the Management of the FTSE EDHEC-Risk Efficient Index Series." 2009. *FTSE EDHEC-Risk Efficient Index Series* (December).

Michaud, Richard. 1989. "The Markowitz Optimization Enigma: Is Optimized Optimal?" *Financial Analysts Journal*, vol. 45, no. 1 (January/February):31–42.

Perold, André. 2007. "Fundamentally Flawed Indexing." *Financial Analysts Journal*, vol. 63, no. 6 (November/December):31–37.

Vuolteenaho, Tuomo. 2002. "What Drives Firm-Level Stock Returns?" *Journal of Finance*, vol. 57, no. 1 (February): 233-264.

Table 1a. Return Characteristics of Annually Rebalanced Global Strategies: 1,000 Stocks, 1987–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
MSCI Global Developed[a]	7.58%	15.65%	0.22	—	—	—	8.36%
Heuristic-Weighting							
Equal-Weighting (EW)	8.64%	15.94%	0.28	1.05%	3.02%	0.35	21.78%
Risk-Clusters EW (k clusters)	10.78%	16.57%	0.40	3.20%	6.18%	0.52	32.33%
Diversity-Weighting ($p = 0.76$)	7.75%	15.80%	0.22	0.16%	1.60%	0.10	10.39%
Fundamentals-Weighting	11.13%	15.30%	0.45	3.54%	4.77%	0.74	14.93%
Optimized-Weighting							
Minimum-Variance	8.59%	11.19%	0.39	1.01%	8.66%	0.12	51.95%
Maximum Diversification	7.77%	13.16%	0.27	0.18%	7.41%	0.02	59.72%
Risk Efficient ($\lambda = 2$)	8.94%	14.90%	0.32	1.35%	3.58%	0.38	36.40%

[a] For the MSCI Global Developed Index, we report turnover of a simulated global developed cap-weighted index of the top 1,000 stocks rebalanced annually on December 31.

Table 1b. Return Characteristics of Annually Rebalanced U.S. Strategies: 1,000 Stocks, 1964–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
S&P 500 [a]	9.46%	15.13%	0.26	—	—	—	6.69%
Heuristic-Weighting							
Equal-Weighting	11.78%	17.47%	0.36	2.31%	6.37%	0.36	22.64%
Risk-Clusters EW (k clusters)	10.91%	14.84%	0.36	1.45%	4.98%	0.29	25.43%
Diversity-Weighting ($p = 0.76$)	10.27%	15.77%	0.30	0.81%	2.63%	0.31	8.91%
Fundamentals-Weighting	11.60%	15.38%	0.39	2.14%	4.50%	0.47	13.60%
Optimized-Weighting							
Minimum-Variance	11.40%	11.87%	0.49	1.94%	8.08%	0.24	48.45%
Maximum Diversification	11.99%	14.11%	0.45	2.52%	7.06%	0.36	56.02%
Risk Efficient ($\lambda = 2$)	12.46%	16.54%	0.42	3.00%	6.29%	0.48	34.19%

[a] For the S&P 500, we report turnover of a simulated U.S. cap-weighted index of the top 500 stocks rebalanced annually on December 31. Actual S&P 500 turnover is generally lower due to committee based stock selection rules.

Figure 1a. Return Characteristics of Annually Rebalanced Global Strategies: 1,000 Stocks, 1987–2009

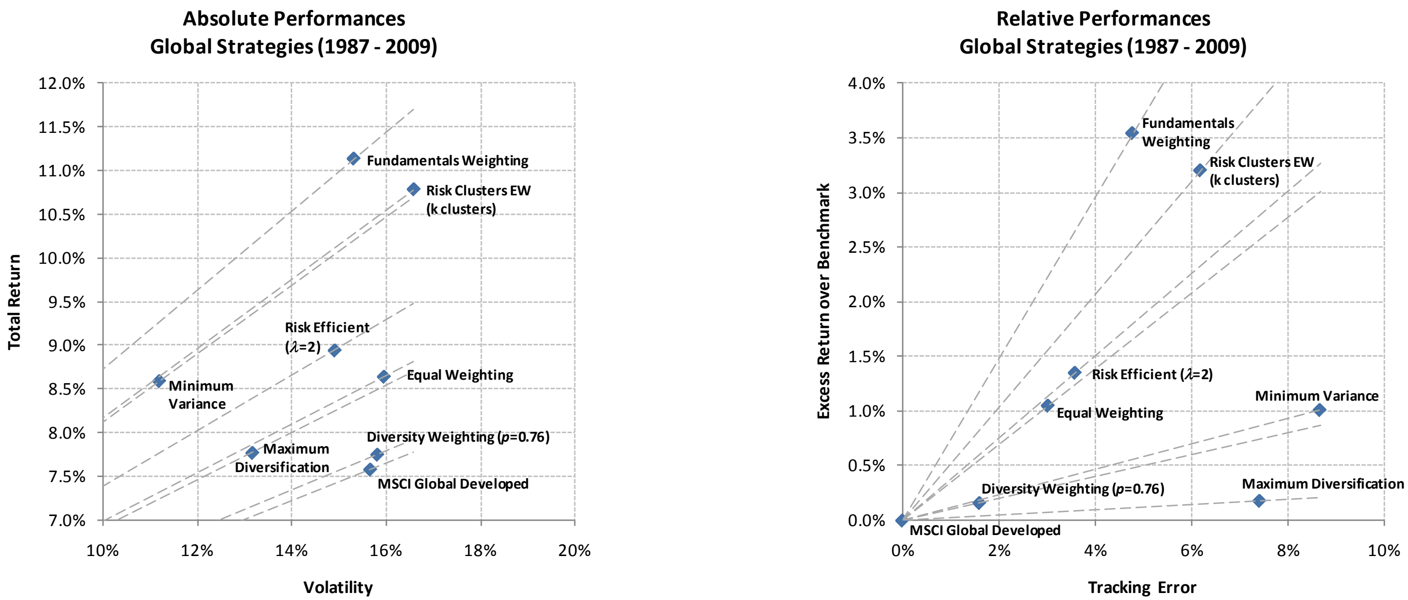

Figure 1b. Return Characteristics of Annually Rebalanced U.S. Strategies: 1,000 Stocks, 1964–2009

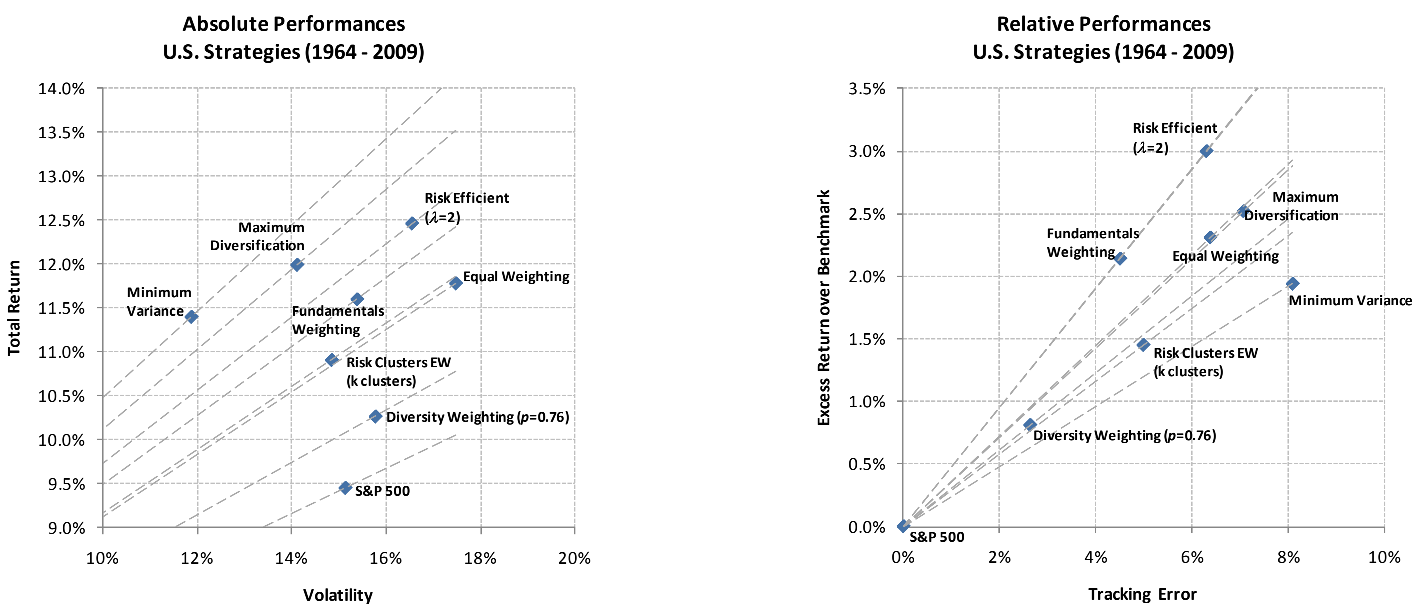

Table 2. Excess Return over Benchmark for Various User Parameters

	Global					United States				
	Annually Rebalanced, 1,000 Stocks	5% Weight Limit	1% Weight Limit	Annually Rebalanced, 500 Stocks	Quarterly Rebalanced, 1,000 Stocks	Annually Rebalanced, 1,000 Stocks	5% Weight Limit	1% Weight Limit	Annually Rebalanced, 500 Stocks	Quarterly Rebalanced, 1,000 Stocks
Heuristic-Weighting										
Equal-Weighting (EW)	1.05%	1.05%	1.05%	0.55%	1.19%	2.31%	2.31%	2.31%	1.26%	2.11%
Risk-Clusters EW (k clusters)	3.20%	3.20%	2.82%	2.86%	3.17%	1.45%	1.45%	1.34%	1.43%	1.29%
Diversity-Weighting (p = 0.76)	0.16%	0.16%	0.21%	-0.24%	0.26%	0.81%	0.81%	0.95%	0.38%	0.75%
Fundamentals-Weighting	3.54%	3.54%	3.50%	3.29%	3.39%	2.14%	2.16%	2.47%	1.84%	1.97%
Optimized-Weighting										
Minimum-Variance	1.01%	1.01%	1.44%	-0.79%	0.38%	1.94%	1.94%	1.80%	1.12%	1.54%
Maximum Diversification	0.18%	0.16%	-0.04%[a]	-0.94%	-0.14%	2.52%	2.53%	2.17%	1.87%	1.82%
Risk Efficient[b]	1.35%	1.02%	1.18%	1.25%	1.32%	3.00%	2.23%	2.36%	2.06%	2.40%

Note: 1987-2009 for Global, 1964-2009 for U.S.

[a] In 5 out of 23 years in the simulation the number of securities with non-zero weights was less than 100, which made the maximum weight exceed 1 percent.

[b] Maximum weight limits of 1 percent and 5 percent were achieved by λ choices of 10 and 50.

Table 3a. Optimization-Based Strategies with Different Covariance Matrix Estimation Methods: Annually Rebalanced Global Portfolios, 1,000 Stocks, 1987–2009

Strategy	Bayesian Shrinkage (A) Excess Return over Benchmark	Bayesian Shrinkage (A) Volatility	Bayesian Shrinkage (A) Tracking Error	Bayesian Shrinkage (B) Excess Return over Benchmark	Bayesian Shrinkage (B) Volatility	Bayesian Shrinkage (B) Tracking Error	Principal Component Analysis Excess Return over Benchmark	Principal Component Analysis Volatility	Principal Component Analysis Tracking Error
Minimum-Variance	1.01%	11.19%	8.66%	1.29%	9.85%	11.00%	0.31%	10.76%	9.42%
Maximum Diversification	1.44%	15.63%	8.45%	0.18%	13.16%	7.41%	–0.40%	13.17%	8.60%
Risk Efficient ($\lambda = 2$)	1.44%	16.39%	3.56%	1.44%	15.69%	3.37%	1.35%	14.90%	3.58%

Table 3b. Optimization-Based Strategies with Different Covariance Matrix Estimation Methods: Annually Rebalanced U.S. Portfolios, 1,000 Stocks, 1964–2009

Strategy	Bayesian Shrinkage (A) Excess Return over Benchmark	Bayesian Shrinkage (A) Volatility	Bayesian Shrinkage (A) Tracking Error	Bayesian Shrinkage (B) Excess Return over Benchmark	Bayesian Shrinkage (B) Volatility	Bayesian Shrinkage (B) Tracking Error	Principal Component Analysis Excess Return over Benchmark	Principal Component Analysis Volatility	Principal Component Analysis Tracking Error
Minimum-Variance	1.94%	11.87%	8.08%	1.54%	11.61%	8.10%	2.25%	11.59%	9.13%
Maximum Diversification	3.72%	17.64%	10.02%	2.52%	14.11%	7.06%	2.92%	14.55%	9.16%
Risk Efficient ($\lambda = 2$)	3.02%	18.19%	7.41%	2.93%	16.87%	6.32%	3.00%	16.54%	6.29%

Table 4a. Four-Factor Model Risk Decomposition of Annually Rebalanced Global Strategies: 1,000 Stocks, 1987–2009

Strategy	Annual Alpha	Alpha p-value	Market (Mkt – R_f)	Size (SMB)	Value (HML)	Momentum (MOM)	R^2
MSCI Global Developed	0.00%	--	1.000	0.000	0.000	0.000	1.00
Equal-Weighting	0.77%	*(0.131)*	1.015†	0.259†	0.025*	-0.008	0.98
Risk-Clusters EW (k clusters)	0.68%	*(0.547)*	1.071†	0.338†	0.232†	0.045†	0.90
Diversity-Weighting (p = 0.76)	0.38%	*(0.173)*	1.001†	0.087†	-0.058†	0.011*	0.99
Fundamentals-Weighting	2.18%	*(0.000)*	0.970†	0.040*	0.332†	-0.090†	0.97
Minimum-Variance	1.25%	*(0.329)*	0.628†	0.001	0.138†	-0.013	0.73
Maximum Diversification	0.49%	*(0.716)*	0.760†	0.097*	0.004	0.029	0.78
Risk Efficient (λ = 2)	0.97%	*(0.154)*	0.947†	0.176*	0.056†	-0.003	0.96

Notes: The MSCI Global Developed was used in the market factor; the HML and SMB factors were simulated following the methodology outlined on French's website with two exceptions: (1) factor portfolios were rebalanced in September to guarantee no look-ahead bias in the global accounting data and (2) instead of the NYSE median breakpoint, we used the top 20 global universe percentile as a cut-off point between the small and large portfolios.

† Significant at 0.01 level

* Significant at 0.10 level

Table 4b. Four-Factor Model Risk Decomposition of Annually Rebalanced U.S. Strategies: 1,000 Stocks, 1964–2009

Strategy	Annual Alpha	Alpha p-value	Market (Mkt – R_f)	Size (SMB)	Value (HML)	Momentum (MOM)	R^2
S&P 500	0.00%	--	1.000	0.000	0.000	0.000	1.00
Equal-Weighting	0.15%	*(0.786)*	1.043†	0.482†	0.144†	-0.012	0.96
Risk-Clusters EW (k clusters)	-0.13%	*(0.846)*	0.954†	0.116†	0.185†	0.040†	0.91
Diversity-Weighting (p = 0.76)	0.07%	*(0.798)*	1.012†	0.173†	0.029†	0.002	0.99
Fundamentals-Weighting	0.50%	*(0.193)*	1.010†	0.128†	0.338†	-0.076†	0.97
Minimum-Variance	0.30%	*(0.713)*	0.708†	0.198†	0.344†	0.011	0.81
Maximum Diversification	-0.02%	*(0.977)*	0.844†	0.342†	0.264†	0.061†	0.87
Risk Efficient (λ = 2)	0.19%	*(0.732)*	1.002†	0.465†	0.250†	0.004	0.95

Note: The S&P 500 was used in the market factor; SMB, HML, and MOM factor portfolios were downloaded from Ken French's website.

† Significant at 0.01 level

Table 5a. Minimum-Variance-Mimicking Portfolios Formed for Annually Rebalanced Global Strategies: 1,000 Stocks, 1987–2009

Strategy	% Invested in Strategy (x)	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio
MSCI Global Developed	71.53%	6.89%	11.19%	0.24	-0.70%	4.45%	-0.16
Heuristic-Weighting							
Equal-Weighting	70.22%	7.60%	11.19%	0.30	0.01%	5.12%	0.00
Risk-Clusters EW (k clusters)	67.54%	8.95%	11.19%	0.42	1.37%	6.71%	0.20
Diversity-Weighting ($p = 0.76$)	70.84%	6.99%	11.19%	0.25	-0.60%	4.65%	-0.13
Fundamentals-Weighting	73.16%	9.49%	11.19%	0.47	1.90%	6.04%	0.32
Optimized-Weighting							
Minimum-Variance	100.00%	8.59%	11.19%	0.39	1.01%	8.66%	0.12
Maximum Diversification	85.02%	7.35%	11.19%	0.28	-0.24%	7.83%	-0.03
Risk Efficient ($\lambda = 2$)	75.11%	7.97%	11.19%	0.34	0.39%	5.39%	0.07

Note: Performances of investing x% in the alternative-beta strategies and 1-x% in U.S. 1-month T-bills.

Table 5b. Minimum-Variance-Mimicking Portfolios Formed for Annually Rebalanced U.S. Strategies: 1,000 Stocks, 1964–2009

Strategy	% Invested in Strategy (x)	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio
S&P 500	78.46%	8.82%	11.87%	0.27	-0.64%	3.26%	-0.20
Heuristic-Weighting							
Equal-Weighting	67.94%	10.12%	11.87%	0.38	0.66%	5.87%	0.11
Risk-Clusters EW (k clusters)	79.97%	10.02%	11.87%	0.38	0.56%	5.51%	0.10
Diversity-Weighting ($p = 0.76$)	75.28%	9.35%	11.87%	0.32	-0.11%	3.94%	-0.03
Fundamentals-Weighting	77.17%	10.42%	11.87%	0.41	0.96%	5.12%	0.19
Optimized-Weighting							
Minimum-Variance	100.00%	11.40%	11.87%	0.49	1.94%	8.08%	0.24
Maximum Diversification	84.11%	11.09%	11.87%	0.47	1.63%	7.19%	0.23
Risk Efficient ($\lambda = 2$)	71.76%	10.79%	11.87%	0.44	1.33%	6.13%	0.22

Note: Performances of investing x% in the alternative-beta strategies and 1-x% in U.S. 1-month T-bills.

Table 6. Transactions Cost Analysis

Strategy	Global						United States					
	Excess Return over Benchmark	One-Way Turnover	Market Cap. USD Bn	Ave. Bid-Ask Spread	Adj. Daily Volume USD Mn	Trading Cost[a, b, c]	Excess Return over Benchmark	One-Way Turnover	Market Cap. USD Bn	Ave. Bid-Ask Spread	Adj. Daily Volume USD Mn	Trading Cost[a, c]
Cap-Weighted Benchmark		8.4%[d]	66.34	0.11%	464.91	0.10%[d]		6.69%[e]	80.80	0.03%	735.40	0.03%[e]
Heuristic-Weighting												
Equal-Weighting	1.05%	21.8%	23.90	0.16%	174.96	0.31%	2.31%	22.6%	11.48	0.06%	132.49	0.22%
Risk-Clusters EW (k	3.20%	32.3%	37.47	0.17%	189.12	0.69%	1.45%	25.4%	37.14	0.04%	312.04	0.12%
Diversity-Weighting (*p* =	0.16%	10.4%	52.37	0.12%	368.16	0.13%	0.81%	8.9%	50.53	0.04%	477.87	0.06%
Fundamentals-Weighting	3.54%	14.9%	59.14	0.14%	397.81	0.28%	2.14%	13.6%	66.26	0.05%	617.47	0.13%
Optimized-Weighting												
Minimum-Variance	1.01%	52.0%	23.97	0.35%	128.43	0.49%	1.94%	48.4%	19.63	0.05%	136.37	0.43%
Maximum Diversification	0.18%	59.7%	20.08	0.45%	122.50	0.57%	2.52%	56.0%	14.77	0.06%	124.08	0.53%
Risk Efficient ($\lambda = 2$)	1.35%	36.4%	26.90	0.15%	193.53	0.33%	3.00%	34.2%	12.06	0.06%	140.07	0.25%

Note: Excess Return, Turnover, and Trading Cost estimated for the period of 1987-2009 for Global, 1964-2009 for U.S.; Market Capitalization, Bid-Ask Spread and Adjusted Daily Volume estimated for rebalancing at the end of 2009.

[a] Estimated with the model suggested by Keim and Madhavan (1997), which accounts for (i) different exchanges, (ii) size of trade, (iii) market-capitalization, (iv) price per share, and (v) type of trade. Portfolio size is fixed at US$100M, type of trade is set as indexing.

[b] The cost model was modified to reflect additional costs for trading through London exchange (50 bps for selling) and Hong Kong exchange (10 bps for buying and selling).

[c] Trading costs include portfolio rebalancing only, not the costs of entering and exiting the strategies.

[d] Turnover and trading cost based on a simulated cap-weighted index of the top 1,000 stocks in the Global Developed market.

[e] Turnover and trading cost based on a simulated cap-weighted index of the top 500 stocks in the U.S. market.

Appendix Ia. Sub-Sample Returns for Global Alternative Equity Indexes: 1,000 Stocks, 1987–2009

Strategy	Since 1987 Total Return	Since 1987 Carhart Four-Factor Alpha	1987 – 89 Total Return	1987 – 89 Carhart Four-Factor Alpha	1990 – 99 Total Return	1990 – 99 Carhart Four-Factor Alpha	2000 – 09 Total Return	2000 – 09 Carhart Four-Factor Alpha
MSCI Global Developed	7.58%	0.00%	19.26%	0.00%	11.96%	0.00%	0.23%	0.00%
Equal-Weighting	8.64%	0.77%	20.35%	-0.05%	10.57%	-0.51%	3.51%	1.92%*
Risk-Clusters EW (k clusters)	10.78%	0.68%	18.64%	-2.93%	13.96%	-0.47%	5.50%	3.86%*
Diversity-Weighting (p = 0.76)	7.75%	0.38%	18.89%	-0.23%	11.39%	0.03%	1.19%	0.82%*
Fundamentals-Weighting	11.13%	2.18%†	20.52%	2.15%	13.87%	1.15%	5.84%	3.05%†
Minimum-Variance	8.59%	1.25%	18.73%	0.74%	8.94%	-0.36%	5.38%	1.67%
Maximum Diversification	7.77%	0.49%	17.00%	-2.26%	8.60%	-1.06%	4.34%	0.85%
Risk Efficient (l = 2)	8.94%	0.97%	20.86%	0.36%	9.73%	-1.01%	4.83%	2.13%*

† Significant at 0.01 level

* Significant at 0.10 level

Appendix Ib. Sub-Sample Returns for U.S. Alternative Equity Indexes: 1,000 Stocks, 1964–2009

Strategy	Since 1964 Total Return	Since 1964 Carhart Four-Factor Alpha	1964 – 69 Total Return	1964 – 69 Carhart Four-Factor Alpha	1970 – 79 Total Return	1970 – 79 Carhart Four-Factor Alpha	1987 – 89 Total Return	1987 – 89 Carhart Four-Factor Alpha	1990 – 99 Total Return	1990 – 99 Carhart Four-Factor Alpha	2000 – 09 Total Return	2000 – 09 Carhart Four-Factor Alpha
MSCI Global Developed	9.46%	0.00%	6.75%	0.00%	5.88%	0.00%	17.55%	0.00%	18.21%	0.00%	-0.95%	0.00%
Equal-Weighting	11.78%	0.15%	14.57%	0.12%	7.77%	-0.68%	17.71%	0.48%	16.26%	-0.80%	4.28%	2.21%
Risk-Clusters EW (k clusters)	10.91%	-0.13%	6.76%	0.04%	8.11%	-0.06%	19.52%	1.13%	12.02%	-3.34%*	6.97%	4.91%†
Diversity-Weighting (p = 0.76)	10.27%	0.07%	9.02%	0.04%	6.46%	-0.16%	17.50%	0.07%	17.61%	-0.18%	1.18%	1.11%*
Fundamentals-Weighting	11.60%	0.50%	7.54%	-0.12%	8.81%	0.11%	19.38%	0.25%	16.89%	0.31%	4.44%	1.23%
Minimum-Variance	11.40%	0.30%	10.31%	0.54%	8.35%	-0.10%	21.00%	1.68%	11.78%	-0.66%	5.73%	1.46%
Maximum Diversification	11.99%	-0.02%	13.63%	0.38%	7.98%	-0.83%	21.01%	1.40%	13.85%	-1.37%	4.80%	1.12%
Risk Efficient (l = 2)	12.46%	0.19%	13.93%	0.26%	9.55%	0.02%	18.52%	0.37%	15.43%	-0.55%	5.92%	2.32%*

† Significant at 0.01 level

* Significant at 0.10 level

Appendix IIa. Comparing Key Alternative Equity Indexes and their Variants, Annually Rebalanced Global Portfolios, 1987–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
MSCI Global Developed	7.58%	15.65%	0.22	--	--	--	9.32%[g]
Equal-Weighting (*N* = 1,000)	8.64%	15.94%	0.28	1.05%	3.02%	0.35	21.78%
Equal-Weighting (*N* = 500)[a]	8.14%	15.89%	0.25	0.55%	2.15%	0.26	22.06%
Risk-Clusters EW (k clusters)	10.78%	16.57%	0.40	3.20%	6.18%	0.52	32.33%
Risk-Clusters EW (k/2 clusters)[b]	10.76%	16.55%	0.40	3.17%	6.42%	0.49	30.16%
Diversity-Weighting (*p* = 0.76)	7.75%	15.80%	0.22	0.16%	1.60%	0.10	10.39%
Diversity-Weighting (*p* = 0.5)[c]	8.16%	15.83%	0.25	0.58%	2.07%	0.28	14.02%
Fundamentals-Weighting (Composite Factors)	11.13%	15.30%	0.45	3.54%	4.77%	0.74	14.93%
Fundamentals-Weighting (Single Unsmoothed Factor)[d]	11.68%	14.82%	0.50	4.10%	6.20%	0.66	19.30%
Minimum-Variance (Bayesian Shrinkage)	8.59%	11.19%	0.39	1.01%	8.66%	0.12	51.95%
Minimum-Variance (PCA)[e]	7.89%	10.76%	0.34	0.31%	9.42%	0.03	56.11%
Maximum Diversification (Bayesian Shrinkage)	7.77%	13.16%	0.27	0.18%	7.41%	0.02	59.72%
Maximum Diversification (PCA)[e]	7.18%	13.17%	0.23	-0.40%	8.60%	-0.05	60.14%
Risk Efficient (λ = 2)	8.94%	14.90%	0.32	1.35%	3.58%	0.38	36.40%
Risk Efficient (λ = 50)[f]	8.60%	13.67%	0.32	1.02%	4.95%	0.21	76.31%

[a] Equally weighting the 1,000 and 500 largest stocks by market-capitalization.

[b] Grouping and equally weighting 20 and 10 risk-clusters of sector portfolios.

[c] Setting the blending factor to 0.76 as chosen by INTECH for their U.S. simulations, and to 0.5 for a stronger tilt toward small-cap and value.

[d] Weighting by a composite of four fundamental factors as defined by Arnott, et al. (2005) – Composite Factors, and by one-year dividend – Single Unsmoothed Factor.

[e] Computing covariance matrix by Bayesian Shrinkage and by Principal Component Analysis

[f] Setting the weight restriction factor λ to 2 as defined by Amenc, et al. (2010), and to 50 to allow maximum single-stock concentration of 5%.

[g] Turnover based on a simulated cap-weighted index.

Appendix IIb. Comparing Key Alternative Equity Indexes and their Variants, Annually Rebalanced U.S. Portfolios, 1964–2009

Strategy	Total Return	Volatility	Sharpe Ratio	Excess Return over Benchmark	Tracking Error	Information Ratio	One-Way Turnover
S&P 500	9.46%	15.13%	0.26	--	--	--	6.69%[g]
Equal-Weighting (N = 1,000)	11.78%	17.47%	0.36	2.31%	6.37%	0.36	22.64%
Equal-Weighting (N = 500)[a]	10.72%	16.48%	0.31	1.26%	4.27%	0.29	20.27%
Risk-Clusters EW (k clusters)	10.91%	14.84%	0.36	1.45%	4.98%	0.29	25.43%
Risk Clusters EW (k/2 clusters)[b]	9.82%	15.55%	0.27	0.36%	4.79%	0.08	29.08%
Diversity-Weighting (p = 0.76)	10.27%	15.77%	0.30	0.81%	2.63%	0.31	8.91%
Diversity-Weighting (p = 0.5)[c]	10.87%	16.37%	0.32	1.41%	4.11%	0.34	13.12%
Fundamentals-Weighting (Composite Factors)	11.60%	15.38%	0.39	2.14%	4.50%	0.47	13.60%
Fundamentals-Weighting (Single Unsmoothed Factor)[d]	10.95%	14.34%	0.38	1.49%	5.18%	0.29	13.53%
Minimum-Variance (Bayesian Shrinkage)	11.40%	11.87%	0.49	1.94%	8.08%	0.24	48.45%
Minimum-Variance (PCA)[e]	11.71%	11.59%	0.53	2.25%	9.13%	0.25	51.68%
Maximum Diversification (Bayesian Shrinkage)	11.99%	14.11%	0.45	2.52%	7.06%	0.36	56.02%
Maximum Diversification (PCA)[e]	12.38%	14.55%	0.47	2.92%	9.16%	0.32	59.91%
Risk Efficient (λ = 2)	12.46%	16.54%	0.42	3.00%	6.29%	0.48	34.19%
Risk Efficient (λ = 50)[f]	11.69%	15.07%	0.41	2.23%	6.33%	0.35	74.21%

[a] Equally weighting the 1,000 and 500 largest stocks by market-capitalization.

[b] Grouping and equally weighting 7 and 4 risk-clusters of sector portfolios.

[c] Setting the blending factor to 0.76 as chosen by INTECH for their U.S. simulations, and to 0.5 for a stronger tilt toward small-cap and value.

[d] Weighting by a composite of four fundamental factors as defined by Arnott, et al. (2005) – Composite Factors, and by one-year dividend – Single Unsmoothed Factor.

[e] Computing covariance matrix by Bayesian Shrinkage and by Principal Component Analysis

[f] Setting the weight restriction factor λ to 2 as defined by Amenc, et al. (2010), and to 50 to allow maximum single-stock concentration of 5%.

[g] Turnover based on a simulated cap-weighted index. Actual S&P 500 turnover is generally lower due to committee based stock selection rules.

Appendix IIIa. Three-Factor Model Risk Decomposition of Annually Rebalanced Global Strategies: 1,000 Stocks, 1987–2009

Strategy	Annual Alpha	Alpha *p*-value	Market ($Mkt - R_f$)	Small-Cap (SMB)	Value (HML)	R^2
MSCI Global Developed	0.00%		1.000	0.000	0.000	1.00
Equal-Weighting	0.68%	*(0.175)*	1.018†	0.260†	0.023*	0.98
Risk-Clusters EW (k clusters)	1.19%	*(0.287)*	1.053†	0.332†	0.241†	0.90
Diversity-Weighting (p = 0.76)	0.51%	*(0.071)*	0.996†	0.085†	–0.056†	0.99
Fundamentals-Weighting	1.15%	*(0.100)*	1.005†	0.053*	0.315†	0.96
Minimum-Variance	1.10%	*(0.384)*	0.633†	0.003	0.135†	0.73
Maximum Diversification	0.82%	*(0.536)*	0.749†	0.093*	0.010	0.78
Risk Efficient (λ = 2)	0.94%	*(0.162)*	0.949†	0.176†	0.055†	0.96

Notes: The MSCI Global Developed is used in the market factor; the HML and SMB factors were simulated following the methodology outlined on French's website with two exceptions: (1) factor portfolios were rebalanced in September to guarantee no look-ahead bias in the global accounting data and (2) instead of the NYSE median breakpoint, we used the top 20 global universe percentile as a cut-off point between the small-cap and big-cap portfolios.

† Significant at 0.01 level

* Significant at 0.10 level

Appendix IIIb. Three-Factor Model Risk Decomposition of Annually Rebalanced U.S. Strategies: 1,000 Stocks, 1964–2009

Strategy	Annual Alpha	Alpha *p*-value	Market ($Mkt - R_f$)	Small-Cap (SMB)	Value (HML)	R^2
S&P 500	0.00%		1.000	0.000	0.000	1.00
Equal-Weighting	0.01%	*(0.984)*	1.046†	0.482†	0.148†	0.96
Risk-Clusters EW (k clusters)	0.33%	*(0.630)*	0.946†	0.115†	0.171†	0.91
Diversity-Weighting (p = 0.76)	0.10%	*(0.718)*	1.012†	0.173†	0.028†	0.99
Fundamentals-Weighting	–0.37%	*(0.369)*	1.026†	0.131†	0.364†	0.97
Minimum-Variance	0.42%	*(0.591)*	0.706†	0.197†	0.341†	0.81
Maximum Diversification	0.68%	*(0.389)*	0.830†	0.340†	0.243†	0.86
Risk Efficient (λ = 2)	0.24%	*(0.659)*	1.001†	0.465†	0.249†	0.95

Note: The S&P 500 was used in the market factor; the SMB and HML factor portfolios were downloaded from Ken French's website.

† Significant at 0.01 level